湖北工業大學
HUBEI UNIVERSITY OF TECHNOLOGY

Course Title	Elementary Number Theory
Course Code	MATH 3041
Semester	Summer 2026
Course Length	4 Weeks, 60 Contact Hours
Credits	4
Instructor	Wang Mei
Office	NO.2 Teaching Building B206
Email	15072447699@163.com
Prerequisite	MATH 1112 Calculus II MATH 2151 Linear Algebra I

Course Description:

This course provides a comprehensive introduction to the properties of integers. The curriculum explores foundational principles of divisibility, the distribution of primes, and the theory of congruences. Students investigate classical number-theoretic functions, the properties of primitive roots, and quadratic residues. The course concludes with an exploration of Diophantine equations and the representation of integers as sums of squares, bridging the gap between historical problems and modern algebraic thought.

Delivery Mode: In-Person, Face-to-Face Instruction

Course Goals:

Students who successfully complete this course will demonstrate competency in the following general education core goals:

- **Critical Thinking Skills** – Students will engage in analytical thinking, demonstrating the ability to critically evaluate, synthesize, and apply knowledge to complex problems, and construct well-reasoned solutions and arguments.
- **Independent Research and Inquiry** – Students will conduct independent research, utilizing academic resources to explore relevant topics, formulating research questions, analyzing data, and presenting findings in a coherent, scholarly manner.
- **Problem-Solving and Application** – Students will apply theoretical concepts and methodologies learned in the course to real-world problems, demonstrating the ability to develop practical solutions informed by academic inquiry.
- **Global and Cultural Awareness** – Students will gain awareness of the global and cultural contexts relevant to the course, appreciating diverse perspectives and considering the implications of their studies in a broader, international context.

Student Learning Outcomes:

Upon completion of this course, students will be able to:

- Apply the Euclidean Algorithm and solve Linear Diophantine Equations;
- Utilize the Chinese Remainder Theorem to solve systems of simultaneous congruences;
- Analyze the properties of arithmetic functions including the Möbius and Euler ϕ-function;
- Evaluate quadratic residues using the Law of Quadratic Reciprocity;
- Prove foundational theorems such as those of Fermat, Euler, and Wilson.

Textbooks/Supplies/Materials/Equipment/ Technology or Technical Requirements:

Primary: *Elementary Number Theory, 7th Edition,* by David M. Burton. McGraw-Hill Higher Education.
Supplemental: *A Friendly Introduction to Number Theory, 4th edition,* by Joseph H. Silverman. Pearson.

Course Requirements:

Problem Sets (30%)
These regular assignments are the core of the learning process. Students transition from computational exercises to writing formal, logical proofs regarding the properties of integers. Emphasis is placed on clarity of mathematical expression and the correct application of theorems like the Fundamental Theorem of Arithmetic.

Unit Quizzes (15%)
Periodic short assessments (roughly 20 minutes) are administered to ensure students are maintaining pace with the modular arithmetic and notation changes.

Midterm Examination (20%)
This written exam evaluates the student's mastery of the first half of the course, specifically divisibility, the Euclidean algorithm, and the structure of congruences. It requires both the execution of algorithms and the derivation of shorter proofs.

Cumulative Final Exam (35%)
A summative assessment covering the entire course. It tests the ability to connect disparate topics, such as using primitive roots in cryptographic contexts or applying quadratic reciprocity to solve complex congruences.

Assessments: Activity	**Percent Contribution**
Problem Sets	30%
Unit Quizzes	15%
Midterm Examination	20%
Cumulative Final Exam	35%

Grading:

Final grades will be based on the sum of all possible course points as noted above.

Grade	**Percentage of available points**

A	94-100
A-	90-93
B+	87-89
B	84-86
B-	80-83
C+	77-79
C	74-76
C-	70-73
D	64-69
D-	60-63
F	0-59

Course Schedule:
The schedule of activities is subject to change at the reasonable discretion of the instructor. Minor changes will be announced in class, major ones provided in writing.

MATH 3041 Schedule

Lecture	Topic	Readings (*Burton*)
L1	Induction, Binomial Theorem & Divisibility Basics	Sections 1.1-1.2
L2	The Division Algorithm	Section 2.2
L3	Prime Numbers	Chapter 3
L4	Greatest Common Divisor	Section 2.3
	Problem Set 1 Due	
L5	The Euclidean Algorithm	Section 2.4
L6	Linear Diophantine Equations	Section 2.5
L7	The Fundamental Theorem of Arithmetic & Prime Distribution	Sections 3.1-3.2
L8	Basic Properties of Congruence	Section 4.2
	Problem Set 2 Due	
L9	Binary/Decimal Representations	Section 4.3
L10	Linear Congruences & Chinese Remainder Theorem	Section 4.4
L11	Midterm Review	Review
	Problem Set 3 Due	
L12	**Midterm Examination**	Chapters 1-4
L13	Fermat's Little Theorem	Section 5.2
L14	Wilson's Theorem	Section 5.3
L15	The Sum and Number of Divisors (σ, τ)	Section 6.1
	Problem Set 4 Due	
L16	The Möbius Inversion Formula	Section 6.2
L17	The Greatest Integer Function	Section 6.3
L18	Euler's Phi-Function (φ)	Section 7.2
L19	Euler's Theorem	Section 7.3
	Problem Set 5 Due	
L20	Order of an Integer & Primitive Roots	Sections 8.1-8.2
L21	Applications to Cryptography	Section 10.3
L22	Euler's Criterion & Legendre Symbol	Sections 9.1-9.2
L23	The Law of Quadratic Reciprocity	Section 9.3
L24	Sums of Squares	Sections 13.1-13.2
	Problem Set 6 Due	
L25	Continued Fractions & Pell's Equation	Chapter 15
	Final Examination	Cumulative

Accommodation Statement:

Academic accommodations may be made for any student who notifies the instructor of the need for an accommodation. It is imperative that you take the initiative to bring such needs to the instructor's attention, as he/she is not legally permitted to inquire. Students who may require assistance in emergency evacuations should contact the instructor as to the most appropriate procedures to follow.

Academic Integrity Statement

Each student is expected to maintain the highest standards of honesty and integrity in academic and professional matters. The University reserves the right to take disciplinary action, up to and including dismissal, against any student who is found guilty of academic dishonesty or otherwise fails to meet the standards. Any student judged to have engaged in academic dishonesty in coursework may receive a reduced or failing grade for the work in question and/or for the course.

Academic dishonesty includes, but is not limited to, dishonesty in quizzes, tests, or assignments; claiming credit for work not done or done by others; hindering the academic work of other students; misrepresenting academic or professional qualifications within or without the University; and nondisclosure or misrepresentation in filling out applications or other University records.

Other Items:

Attendance and Expectations

All students are required to attend every class, except in cases of illness, serious family concerns, or other major problems. We expect that students will arrive on time, be prepared to listen and participate as appropriate, and stay for the duration of a meeting rather than drift in or out casually. In short, we anticipate that students will show professors and fellow students maximum consideration by minimizing the disturbances that cause interruptions in the learning process. This means that punctuality is a must, that cellular phones be turned off, and that courtesy is the guiding principle in all exchanges among students and faculty. You will be responsible for the materials and ideas presented in the lecture.

Assignment Due Dates

All written assignments must be turned in at the time specified. Late assignments will not be accepted unless prior information has been obtained from the instructor. If you believe you have extenuating circumstances, please contact the instructor as soon as possible.

Make-Up Work

The instructor will not provide students with class information or make-up assignments/quizzes/exams missed due to an unexcused absence. Absences will be excused and assignments/quizzes/exams may be made up only with written documentation of an authorized absence. Every effort should be made to avoid scheduling appointments during class. An excused student is responsible for

requesting any missed information from the instructor and setting up any necessary appointments outside of class.

Access, Special Needs, and Disabilities
Please notify the instructor at the start of the semester if you have any documented disabilities, a medical issue, or any special circumstances that require attention, and the school will be happy to assist.