



湖北工业大学
HUBEI UNIVERSITY OF TECHNOLOGY

Course Title	Introduction to Group Theory
Course Code	MATH 3221
Semester	Summer 2026
Course Length	4 Weeks, 60 Contact Hours
Credits	4
Instructor	TBA
Office	TBA
Email	TBA
Prerequisite	MATH 1112 Calculus II MATH 2151 Linear Algebra I

Course Description:

Group theory provides the precise mathematical framework for studying symmetry in geometry, physics, chemistry, and abstract structures. Students will transition from concrete mathematical objects to abstract reasoning by exploring the fundamental properties of groups, including symmetry, permutations, and matrix representations. The course covers the core hierarchy of group theory: from basic axioms and subgroups to sophisticated decomposition theorems such as the Isomorphism and Sylow Theorems. By the end of the term, students will be able to apply group actions to solve problems in geometry and combinatorics.

Course Goals:

Students who successfully complete this course will demonstrate competency in the following general education core goals:

- **Critical Thinking Skills** – Students will engage in analytical thinking, demonstrating the ability to critically evaluate, synthesize, and apply knowledge to complex problems, and construct well-reasoned solutions and arguments.
- **Independent Research and Inquiry** – Students will conduct independent research, utilizing academic resources to explore relevant topics, formulating research questions, analyzing data, and presenting findings in a coherent, scholarly manner.
- **Problem-Solving and Application** – Students will apply theoretical concepts and methodologies learned in the course to real-world problems, demonstrating the ability to develop practical solutions informed by academic inquiry.
- **Global and Cultural Awareness** – Students will gain awareness of the global and cultural contexts relevant to the course, appreciating diverse perspectives and considering the implications of their studies in a broader, international context.

Student Learning Outcomes:

Upon completion of this course, students will be able to:

- State the group axioms, verify them for concrete and abstract examples, and work with permutation groups, matrix groups, and other standard constructions;
- Understand homomorphisms and quotients of groups, as well as group actions on a set, orbits and stabilizers, conjugacy, and be able to determine when a group has a normal subgroup;
- State and apply the Sylow theorems to determine the structure of finite groups and solve related problems;
- Reason mathematically, write proofs, and judge whether an attempted proof in group theory is correct/complete.

Textbooks/Supplies/Materials/Equipment/ Technology or Technical Requirements:

Joseph Gallian, *Contemporary Abstract Algebra, 10th edition*, Chapman & Hall.

Course Requirements:

Homework Assignments (30%): Regular problem sets drawn primarily from the textbook will develop proof-writing skills and conceptual understanding. Each assignment emphasizes clear logical reasoning and complete solutions.

In-Class Quizzes (10%): Short, unannounced or announced quizzes (approximately 4-5 during the semester) will test mastery of definitions, basic examples, and quick applications covered in recent lectures.

Midterm Examination (25%): A 120-minute scheduled midterm will cover material from the first half of the course. It will include a mix of computational problems, proofs, and conceptual questions to evaluate both technical proficiency and deeper understanding.

Final Examination (35%): A comprehensive 180-minute final examination will cover the entire course with emphasis on the latter topics (group actions, Sylow theorems, and synthesis of all concepts). It will feature a balance of routine calculations, non-routine proofs, and applications to ensure students can integrate all learned material.

Assessments: Activity	Percent Contribution
Homework Assignments	30%
In-Class Quizzes	10%
Midterm Examination	25%
Final Examination	35%

Grading:

Final grades will be based on the sum of all possible course points as noted above.

Grade	Percentage of available points
A	94-100
A-	90-93
B+	87-89

B	84-86
B-	80-83
C+	77-79
C	74-76
C-	70-73
D	64-69
D-	60-63
F	0-59

Course Schedule:

The schedule of activities is subject to change at the reasonable discretion of the instructor. Minor changes will be announced in class, major ones provided in writing.

MATH 3221 Schedule		
Lecture	Topic	Readings
L1	Introduction to symmetry and group theory	Ch. 1
L2	Definition and basic properties of groups	Ch. 2
L3	Examples: permutation and matrix groups	Ch. 2 & 5
L4	Subgroups and subgroup tests Homework 1 due	Ch. 3
L5	Generators, cyclic groups	Ch. 4
L6	Order of elements and classification	Ch. 4
L7	Permutation groups and symmetric group	Ch. 5
L8	Alternating group and sign homomorphism Homework 2 due	Ch. 5
L9	Dihedral groups and geometric symmetries	Ch. 1
L10	Homomorphisms - definitions and examples	Ch. 10
L11	Isomorphisms and Cayley's theorem	Ch. 6
L12	Conjugation and conjugacy classes Homework 3 due	Ch. 6
L13	Cosets	Ch. 7
L14	Lagrange's theorem and applications	Ch. 7
---	Midterm Examination	Covers L1-13
L15	Normal subgroups	Ch. 9
L16	Quotient (factor) groups Homework 4 due	Ch. 9
L17	First isomorphism theorem	Ch. 10
L18	Correspondence theorem and further isomorphism theorems	Ch. 9-10
L19	Automorphisms	Ch. 6
L20	Group actions on sets - definitions and examples Homework 5 due	Ch. 27
L21	Orbits, stabilizers, orbit-stabilizer theorem	Ch. 27
L22	p-groups and introduction to Sylow theorems	Ch. 23
L23	Sylow theorems and applications Homework 6 due	Ch. 23
L24	Direct and semidirect products	Ch. 8 & 9
L25	Review and advanced examples/applications	Selected sections from all chapters
---	Final Examination	Cumulative

Accommodation Statement:

Academic accommodations may be made for any student who notifies the instructor of the need for an accommodation. It is imperative that you take the initiative to bring such needs to the instructor's attention, as he/she is not legally permitted to inquire. Students who may require assistance in emergency evacuations should contact the instructor as to the most appropriate procedures to follow.

Academic Integrity Statement

Each student is expected to maintain the highest standards of honesty and integrity in academic and professional matters. The University reserves the right to take disciplinary action, up to and including dismissal, against any student who is found guilty of academic dishonesty or otherwise fails to meet the standards. Any student judged to have engaged in academic dishonesty in coursework may receive a reduced or failing grade for the work in question and/or for the course.

Academic dishonesty includes, but is not limited to, dishonesty in quizzes, tests, or assignments; claiming credit for work not done or done by others; hindering the academic work of other students; misrepresenting academic or professional qualifications within or without the University; and nondisclosure or misrepresentation in filling out applications or other University records.

Other Items:**Attendance and Expectations**

All students are required to attend every class, except in cases of illness, serious family concerns, or other major problems. We expect that students will arrive on time, be prepared to listen and participate as appropriate, and stay for the duration of a meeting rather than drift in or out casually. In short, we anticipate that students will show professors and fellow students maximum consideration by minimizing the disturbances that cause interruptions in the learning process. This means that punctuality is a must, that cellular phones be turned off, and that courtesy is the guiding principle in all exchanges among students and faculty. You will be responsible for the materials and ideas presented in the lecture.

Assignment Due Dates

All written assignments must be turned in at the time specified. Late assignments will not be accepted unless prior information has been obtained from the instructor. If you believe you have extenuating circumstances, please contact the instructor as soon as possible.

Make-Up Work

The instructor will not provide students with class information or make-up assignments/quizzes/exams missed due to an unexcused absence. Absences will be excused and assignments/quizzes/exams may be made up only with written documentation of an authorized absence. Every effort should be made to avoid scheduling appointments during class. An excused student is responsible for

requesting any missed information from the instructor and setting up any necessary appointments outside of class.

Access, Special Needs, and Disabilities

Please notify the instructor at the start of the semester if you have any documented disabilities, a medical issue, or any special circumstances that require attention, and the school will be happy to assist.