湖北工業大學
HUBEI UNIVERSITY OF TECHNOLOGY

Course Title	Real Mathematical Analysis
Course Code	MATH 4013
Semester	Summer 2026
Course Length	8 Weeks, 60 Contact Hours
Credits	4
Instructor	Wang Mei
Office	NO.2 Teaching Building B206
Email	15072447699@163.com
Prerequisite	MATH 3370 Introductory Real Analysis
Antirequisite	MATH 4011 Real Mathematical Analysis (4 Weeks)

Course Description:

This course offers a rigorous study of the theoretical foundations of multivariable calculus and the topology of Euclidean spaces. We will transition from the properties of the real line to the analysis of mappings between higher-dimensional spaces. Key themes include the completeness of metric spaces, the topological nature of continuity, the contraction mapping principle, and the local behavior of differentiable functions. The course concludes with an introduction to the existence and uniqueness theory for differential equations.

Delivery Mode: In-Person, Face-to-Face Instruction

Course Goals:

Students who successfully complete this course will demonstrate competency in the following general education core goals:

- **Critical Thinking Skills** – Students will engage in analytical thinking, demonstrating the ability to critically evaluate, synthesize, and apply knowledge to complex problems, and construct well-reasoned solutions and arguments.
- **Independent Research and Inquiry** – Students will conduct independent research, utilizing academic resources to explore relevant topics, formulating research questions, analyzing data, and presenting findings in a coherent, scholarly manner.
- **Problem-Solving and Application** – Students will apply theoretical concepts and methodologies learned in the course to real-world problems, demonstrating the ability to develop practical solutions informed by academic inquiry.
- **Global and Cultural Awareness** – Students will gain awareness of the global and cultural contexts relevant to the course, appreciating diverse perspectives and considering the implications of their studies in a broader, international context.

Student Learning Outcomes:

Upon completion of this course, students will be able to:

- Work rigorously in metric spaces, proving properties of compactness (Heine-Borel), connectedness, and the Baire category theorem;
- Distinguish pointwise vs. uniform convergence, prove consequences (continuity, integration, differentiation), and apply the Arzelà-Ascoli and Weierstrass approximation theorems;
- Rigorous demonstrate the Inverse and Implicit Function Theorems using the Banach Fixed Point Theorem;
- Understand introductory Banach and Hilbert spaces, linear functionals, and the Hahn-Banach theorem (statement and basic applications);
- Analyze Fourier series, including basic convergence properties;
- Analyze the existence and uniqueness of solutions to systems of ODEs via Picard-Lindelöf.

Textbooks/Supplies/Materials/Equipment/ Technology or Technical Requirements:
Required: *Principles of Mathematical Analysis (3rd Ed.)* by Walter Rudin. McGraw Hill.
Supplemental: *Analysis II (3rd Ed.)* by Terence Tao. Springer.

Course Requirements:

Problem Sets (40%)
These assignments are the core of the learning process. They require students to construct formal proofs regarding topological properties and functional limits. Success requires moving beyond computation to logical derivation.

Midterm Examination (25%)
This exam evaluates the first half of the course, including metric/topology, uniform convergence, function spaces, approximation, introductory Banach/Hilbert/linear functionals.

Final Examination (35%)
A comprehensive final focusing heavily on the second half of the course (Fourier series, multivariable differentiation, contraction mapping, inverse/implicit theorems, ODE applications).

Assessments: Activity	**Percent Contribution**
Problem Sets	40%
Midterm Exam	25%
Final Exam	35%

Grading:

Final grades will be based on the sum of all possible course points as noted above.

Grade	**Percentage of available points**
A	94-100
A-	90-93
B+	87-89

B	84-86
B-	80-83
C+	77-79
C	74-76
C-	70-73
D	64-69
D-	60-63
F	0-59

Course Schedule:
The schedule of activities is subject to change at the reasonable discretion of the instructor. Minor changes will be announced in class, major ones provided in writing.

MATH 4013 Schedule		
Lecture	Topic	Readings
L1	Metric spaces: definitions, examples, basic point-set topology	*Rudin* Ch 2; *Tao* 1.1-1.3
L2	Completeness, Cauchy sequences, contraction mapping & Banach fixed point theorem	*Tao* 1.4, 6.6
L3	Compact metric spaces, Heine-Borel theorem, Bolzano-Weierstrass	*Rudin* Ch 2; *Tao* 1.5
L4	Connectedness, path-connectedness, product spaces	*Rudin* Ch 2; *Tao* 2.4
L5	Continuity in metric spaces, uniform continuity, Heine-Cantor theorem	*Rudin* Ch 4; *Tao* 2.1-2.3
L6	Baire category theorem & applications	*Rudin* Ch 3; *Tao* 1.4
L7	Sequences & series of functions: pointwise vs. uniform convergence	*Rudin* Ch 7; *Tao* 3.1-3.2
L8	Uniform convergence: preservation of continuity, integration, differentiation	*Rudin* Ch 7; *Tao* 3.3, 3.6-3.7
L9	Weierstrass M-test; uniform approximation by polynomials (Weierstrass/Stone-Weierstrass)	*Rudin* Ch 7; *Tao* 3.5, 3.8
L10	Arzelà-Ascoli theorem & equicontinuity in function spaces	*Rudin* Ch 7
L11	Introduction to normed linear spaces & Banach spaces	Supplemental notes
L12	**Midterm Examination**	---
L13	Hilbert spaces: inner products, completeness, orthonormal bases	Supplemental notes
L14	Bounded linear functionals, dual spaces, Hahn-Banach theorem (statement & applications)	Supplemental notes
L15	Fourier series: periodic functions, inner products on periodic functions	*Tao* Ch 5.1-5.3
L16	Trigonometric polynomials, periodic convolutions	*Tao* 5.4
L17	Fourier theorems, Plancherel theorem, convergence properties	*Tao* 5.5
L18	Functions of several variables: linear transformations	*Rudin* Ch 9; *Tao* 6.1
L19	Partial derivatives, directional derivatives, gradients	*Rudin* Ch 9; *Tao* 6.2-6.3
L20	Total (Fréchet) differentiability, chain rule, mean value theorems	*Rudin* Ch 9; *Tao* 6.4
L21	Higher-order partials, Clairaut's theorem	*Rudin* Ch 9; *Tao* 6.5
L22	Contraction mapping applications in multivariable setting	*Rudin* Ch 9; *Tao* 6.6

L23	Inverse function theorem: proof & applications	*Rudin* Ch 9; *Tao* 6.7
L24	Implicit function theorem (Dini's formulation)	*Rudin* Ch 9; *Tao* 6.8
L25	Applications to ODE initial value problems: Picard-Lindelöf theorem	*Rudin* Ch 9 / supplemental
/	**Final Examination**	Cumulative Exam

Accommodation Statement:

Academic accommodations may be made for any student who notifies the instructor of the need for an accommodation. It is imperative that you take the initiative to bring such needs to the instructor's attention, as he/she is not legally permitted to inquire. Students who may require assistance in emergency evacuations should contact the instructor as to the most appropriate procedures to follow.

Academic Integrity Statement

Each student is expected to maintain the highest standards of honesty and integrity in academic and professional matters. The University reserves the right to take disciplinary action, up to and including dismissal, against any student who is found guilty of academic dishonesty or otherwise fails to meet the standards. Any student judged to have engaged in academic dishonesty in coursework may receive a reduced or failing grade for the work in question and/or for the course.

Academic dishonesty includes, but is not limited to, dishonesty in quizzes, tests, or assignments; claiming credit for work not done or done by others; hindering the academic work of other students; misrepresenting academic or professional qualifications within or outside the University; and nondisclosure or misrepresentation in filling out applications or other University records.

Other Items:

Attendance and Expectations

All students are required to attend every class, except in cases of illness, serious family concerns, or other major problems. We expect that students will arrive on time, be prepared to listen and participate as appropriate, and stay for the duration of a meeting rather than drift in or out casually. In short, we anticipate that students will show professors and fellow students maximum consideration by minimizing the disturbances that cause interruptions in the learning process. This means that punctuality is a must, that cellular phones be turned off, and that courtesy is the guiding principle in all exchanges among students and faculty. You will be responsible for the materials and ideas presented in the lecture.

Assignment Due Dates

All written assignments must be turned in at the time specified. Late assignments will not be accepted unless prior information has been obtained from the instructor. If you believe you have extenuating circumstances, please contact the instructor as soon as possible.

Make-Up Work

The instructor will not provide students with class information or make-up assignments/quizzes/exams missed due to an unexcused absence. Absences will be excused and assignments/quizzes/exams may be made up only with written documentation of an authorized absence. Every effort should be made to avoid scheduling appointments during class. An excused student is responsible for requesting any missed information from the instructor and setting up any necessary appointments outside of class.

Access, Special Needs, and Disabilities
Please notify the instructor at the start of the semester if you have any documented disabilities, a medical issue, or any special circumstances that require attention, and the school will be happy to assist.