湖北工業大學
HUBEI UNIVERSITY OF TECHNOLOGY

Course Title	Numerical Linear Algebra
Course Code	MATH 4102
Semester	Summer 2026
Course Length	4 Weeks, 60 Contact Hours
Credits	4
Instructor	TBA
Office	TBA
Email	TBA
Prerequisite	CMPT 1011 Introduction to Computer Science MATH 2231 Multivariable Calculus MATH 2401 Linear Algebra and Matrix Theory

Course Description:

This course introduces the theory and computational methods of modern numerical linear algebra, with emphasis on matrix algorithms arising in scientific computing, data analysis, and large-scale engineering applications. Topics include orthogonalization methods, QR factorization, least-squares problems, singular value decomposition, low-rank approximation, conditioning and backward stability, eigenvalue computations, sparse matrix techniques, and iterative methods for solving large linear systems.

Special attention is given to the relationship between mathematical theory and practical algorithm design, including numerical stability, computational efficiency, and implementation considerations. Students will also study direct and iterative solvers for sparse systems, including Krylov subspace methods. The course serves as a bridge between theoretical linear algebra and computational mathematics and prepares students for advanced study in applied mathematics, scientific computing, machine learning, optimization, and engineering computation.

Course Goals:

Students who successfully complete this course will demonstrate competency in the following general education core goals:

- **Critical Thinking Skills** – Students will engage in analytical thinking, demonstrating the ability to critically evaluate, synthesize, and apply knowledge to complex problems, and construct well-reasoned solutions and arguments.
- **Independent Research and Inquiry** – Students will conduct independent research, utilizing academic resources to explore relevant topics, formulating research questions, analyzing data, and presenting findings in a coherent, scholarly manner.

- **Problem-Solving and Application** – Students will apply theoretical concepts and methodologies learned in the course to real-world problems, demonstrating the ability to develop practical solutions informed by academic inquiry.
- **Global and Cultural Awareness** – Students will gain awareness of the global and cultural contexts relevant to the course, appreciating diverse perspectives and considering the implications of their studies in a broader, international context.

Student Learning Outcomes:

Upon completion of this course, students will be able to:

- Analyze and solve linear systems using direct and iterative numerical methods;
- Apply QR factorization and orthogonalization techniques to least-squares problems;
- Understand and compute singular value decomposition and low-rank approximations;
- Evaluate numerical stability, conditioning, and error propagation in matrix computations;
- Compute eigenvalues and eigenvectors using practical numerical algorithms;
- Understand sparse matrix structures and the challenges of large-scale computation;
- Implement standard numerical linear algebra algorithms using scientific computing software;
- Interpret numerical results with mathematical rigor and computational awareness.

Textbooks/Supplies/Materials/Equipment/ Technology or Technical Requirements:

Textbooks:

Primary: *Numerical Linear Algebra* by Lloyd N. Trefethen and David Bau III (SIAM, 1997).
Supplemental: *Linear Algebra with Applications, 2nd Edition* by Jeffrey Holt (W.H. Freeman & Company, 2017).

Technology Requirements:

Students are required to use MATLAB or Python (NumPy/SciPy) for computational assignments.

Course Requirements:

Problem Sets

Students complete four written assignments distributed throughout the semester, covering both theoretical derivation and analytical problem-solving. The assignments emphasize orthogonality, QR factorization, least-squares systems, conditioning, eigenvalue methods, and iterative solvers. Some problems require short proofs, algorithm justification, and interpretation of numerical behavior.

Computational Lab Reports

Two laboratory reports require students to implement selected numerical linear algebra algorithms using MATLAB or Python (NumPy/SciPy). Typical tasks include QR factorization, least-squares fitting, singular value decomposition, power iteration, and iterative methods such as Conjugate Gradient or GMRES. Each report should include code, computational output, convergence observations, and a short technical discussion.

Midterm Examination
The midterm exam covers the first half of the course, with focus on orthogonality, projections, QR factorization, least-squares problems, singular value decomposition, and numerical conditioning.

Applied Numerical Project
Students complete an individual applied project involving a practical matrix computation problem drawn from scientific computing or data analysis. Possible topics include image compression using SVD, low-rank approximation, PageRank-type eigenvalue problems, sparse matrix solution strategies, or iterative solver comparison. The project requires implementation, mathematical explanation, interpretation of results, and a concise written report demonstrating independent analysis.

Final Examination
The final exam is cumulative, with stronger emphasis on eigenvalue algorithms, Hessenberg reduction, sparse direct methods, Krylov subspace methods, Conjugate Gradient, GMRES, and introductory preconditioning strategies.

All examinations including the midterm and the final exam are conducted in person during scheduled class periods or official examination sessions.

Assessments: Activity	**Percent Contribution**
Problem Sets (4)	20%
Computational Lab Reports (2)	15%
Midterm Examination	20%
Applied Numerical Project	15%
Final Examination	30%

Grading:

Final grades will be based on the sum of all possible course points as noted above.

Grade	**Percentage of available points**
A	94-100
A-	90-93
B+	87-89
B	84-86
B-	80-83
C+	77-79
C	74-76
C-	70-73
D	64-69
D-	60-63
F	0-59

Course Schedule:
The schedule of activities is subject to change at the reasonable discretion of the instructor. Minor changes will be announced in class, major ones provided in writing.

	MATH 4102 Schedule	
Lecture	Topic	Readings
L1	Review of vector spaces, matrix operations, subspaces	*Holt* Ch. 2, Ch. 4, Ch. 7
L2	Orthogonality, inner products, orthogonal projections	*Trefethen* Lecture 2; *Holt* Ch. 8.1, 10.1
L3	Vector norms, matrix norms, induced norms	*Trefethen* Lecture 3
L4	Projectors and least-squares motivation	*Trefethen* Lecture 6; *Holt* Ch. 8.2
L5	QR factorization and geometric interpretation *Problem Set 1 Assigned	*Trefethen* Lecture 7; *Holt* Ch. 8.3
L6	Classical and Modified Gram-Schmidt	*Trefethen* Lecture 8; *Holt* Ch. 8.2, 10.2
L7	Householder triangularization	*Trefethen* Lecture 10
L8	Least-squares problems and normal equations	*Trefethen* Lecture 11; *Holt* Ch. 8.5
L9	Singular Value Decomposition I: theory	*Trefethen* Lecture 4; *Holt* Ch. 8.4
L10	Singular Value Decomposition II: pseudoinverse and low-rank approximation *Problem Set 2 Assigned	*Trefethen* Lecture 5; *Trefethen* Lecture 31
L11	Conditioning and condition numbers *Lab Report 1 Assigned	*Trefethen* Lecture 12
L12	Floating point arithmetic and backward stability	*Trefethen* Lectures 13-14
L13	Stability of QR and least-squares problems	*Trefethen* Lectures 16, 18, 19
L14	**Midterm Examination**	
L15	Gaussian elimination and LU factorization review	*Trefethen* Lecture 20; *Holt* Ch. 3.4
L16	Pivoting strategies and numerical stability *Problem Set 3 Assigned	*Trefethen* Lectures 21-22
L17	Eigenvalue problems and spectral properties	*Trefethen* Lecture 24; *Holt* Ch. 6.1-6.2
L18	Overview of eigenvalue algorithms	*Trefethen* Lecture 25
L19	Hessenberg and tridiagonal reduction	*Trefethen* Lecture 26
L20	Rayleigh quotient and inverse iteration *Problem Set 4 Assigned	*Trefethen* Lecture 27
L21	QR algorithm for eigenvalue computation *Lab Report 2 Assigned	*Trefethen* Lectures 28-29
L22	Sparse matrices and overview of iterative methods	*Trefethen* Lecture 32; supplemental notes
L23	Arnoldi iteration, GMRES	*Trefethen* Lectures 33-35

L24	Lanczos iteration and Conjugate Gradient method *Applied Project Due	*Trefethen* Lectures 36-38
L25	Preconditioning, CG refinement, final review	*Trefethen* Lectures 39-40
/	**Final Examination (Cumulative)**	/

Accommodation Statement:

Academic accommodations may be made for any student who notifies the instructor of the need for an accommodation. It is imperative that you take the initiative to bring such needs to the instructor's attention, as he/she is not legally permitted to inquire. Students who may require assistance in emergency evacuations should contact the instructor as to the most appropriate procedures to follow.

Academic Integrity Statement

Each student is expected to maintain the highest standards of honesty and integrity in academic and professional matters. The University reserves the right to take disciplinary action, up to and including dismissal, against any student who is found guilty of academic dishonesty or otherwise fails to meet the standards. Any student judged to have engaged in academic dishonesty in coursework may receive a reduced or failing grade for the work in question and/or for the course.

Academic dishonesty includes, but is not limited to, dishonesty in quizzes, tests, or assignments; claiming credit for work not done or done by others; hindering the academic work of other students; misrepresenting academic or professional qualifications within or outside the University; and nondisclosure or misrepresentation in filling out applications or other University records.

Other Items:

Attendance and Expectations

All students are required to attend every class, except in cases of illness, serious family concerns, or other major problems. We expect that students will arrive on time, be prepared to listen and participate as appropriate, and stay for the duration of a meeting rather than drift in or out casually. In short, we anticipate that students will show professors and fellow students maximum consideration by minimizing the disturbances that cause interruptions in the learning process. This means that punctuality is a must, that cellular phones be turned off, and that courtesy is the guiding principle in all exchanges among students and faculty. You will be responsible for the materials and ideas presented in the lecture.

Assignment Due Dates

All written assignments must be turned in at the time specified. Late assignments will not be accepted unless prior information has been obtained from the instructor. If you believe you have extenuating circumstances, please contact the instructor as soon as possible.

Make-Up Work

The instructor will not provide students with class information or make-up assignments/quizzes/exams missed due to an unexcused absence. Absences will be excused and assignments/quizzes/exams may be made up only with written documentation of an authorized absence. Every effort should be made to avoid scheduling appointments during class. An excused student is responsible for requesting any missed information from the instructor and setting up any necessary appointments outside of class.

Access, Special Needs, and Disabilities
Please notify the instructor at the start of the semester if you have any documented disabilities, a medical issue, or any special circumstances that require attention, and the school will be happy to assist.